**Make wise use of every drop**

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**Abstract**: **We attributed water scarcity to uneven distribution in space and time and imbalance between supply and demand. We solved the former by transferring water across regions and storing water for future use, and for the latter we considered supply augmentation and demand constraint methods. We first used a grey model to predict gap between demand and supply in 2015. Results are that there will be 15 provinces in short of water. Jiangsu province, the most severe case, will be faced with 58.32 billion of water shortage. Next we developed four models to address water transfer, water storage, desalinization and water conservation. A transportation model was applied to determine an optimal transfer strategy. Results suggested that we transport 12.32 billion of water from the Songliao Region to the Haihe Region, and 5.2 billion of water from the Long River to the Yellow River. We applied a news-vendor model to determine optimal amount of water needed. A case study of Three Gorges Reservoir revealed that 84.1 billion of water should be stored now to satisfy water demand in 2025. A NPV analysis of desalinization projects indicated that 4 desalinization plants should be built in Shanghai, and several more in other provinces in need of water. A Ramsey pricing model was used to determine an optimal pricing strategy. A case study of Shaanxi province revealed that increasing block tariffs achieves a demand reduction of 17.8 per person per year. Finally, we provided a guide for government to make decisions and propose specific measures for four representative regions. Our models are conceptual ones and solutions are based on mathematical optimization. So with more precise data we are able to modify our results without much burdensome repetitions.**

**1 Introduction**

Recent changes in population and geography, from urbanization to climate change, have increased the demand for water and, at the same, degraded water supplies. The issue is even more severe in China. The Dow’s report (2011) [1] pointed out that among 661 cities in China, 33% are short of water, and 17% are regarded as badly in lack of water. Feeding the world's 20% population with the world's 6% total water resources poses a great challenge for China, which is now plagued by uneven distribution of water in space and time. Home to 40% of the population of China, northern regions hold only 5% of the nation’s water resources. Over-withdrawals of surface water and groundwater has led to depletion of water resources and environmental damage in some regions (Oelkers et al., 2011) [2], further exacerbating the issue. So it is urgent for us to take actions to deal with water shortage, reflected both in quantity and quality.

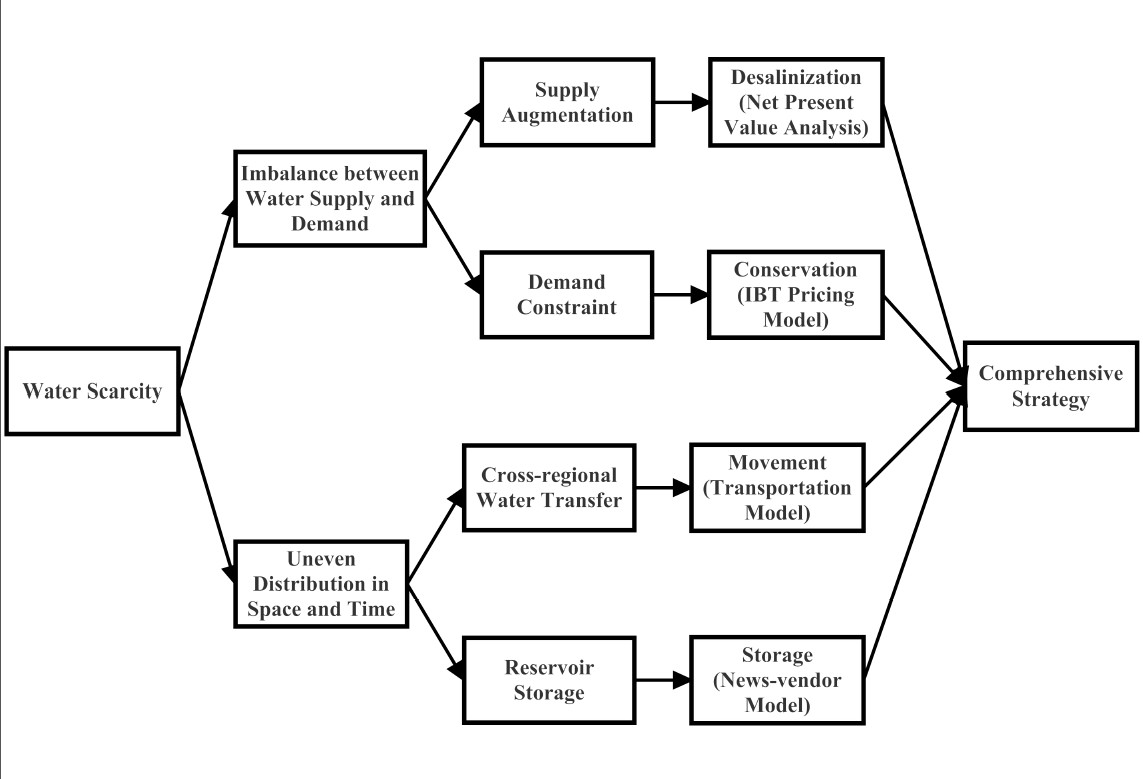
The purpose of the paper is to meet the challenge of sustainable water management in China. We regard the problem as composed of two major kinds, namely imbalance between water supply and demand, as well as uneven water distribution in time and space. First we separately consider four strategies: movement, storage, desalinization and conservation using a transportation model, a stock model, a NPV analysis and an IBT pricing model respectively (see **Figure 1**). Then we synthesize these four strategies from the perspective of decision makers in China. In particular, we answer the following questions:

**1. What is the estimated water demand and available water supply in 2025?** The answer will lead us to the gaps between demand and supply across China. Based on this gap, we can further analyse the problem.

**2. How to solve the predicted problem of water shortage?** Based on the logic of four strategies, we come up with four models tackling with water shortage seperately, and then a comprehensive model will be used for decision makers of China.

For the rest of our paper, we take every province in mainland China, as well as big municipalities like Beijing, into consideration. For simplicity, we will refer to all of them as "provinces" of China, and there are 30 provinces in our case. Hainan province, Hongkong SAR, Macao SAR and Taiwan district are omitted since they are more isolated to water management system of mainland China.

Figure 1: Basic Logic of Our Model



**2 Water Demand and Supply Prediction**

Many methods can be used to predict such time series data as water demand and supply, including auto regression, moving average, Box-Jenkins models, grey models, neural network and so on. As superiority to conventional statistical models, grey models require only a limited amount of data to estimate the behavior of unknown systems (Deng, 1989; Zhang and Liu, 2001) [3] [4]. Therefore, considering limited data available in China, we use a grey model to predict water demand and supply.

**2.1 Data Description**

The original province level data are adapted from 1999 to 2011 by National Bureau of Statistics of China [5].

**We use water usage (in 100 million ) as water demand,** including agriculture, industry, urban consumption and ecological protection. Note that data for year 2004 data are unavailable due to unknown reasons. For consistency, we take the average demand of 2003 and 2005 to substitute for that of 2004.

**We use 40% of total water resource (in million ) as a proxy for water supply.** The total amount of water resource is the sum of surface water resources and groundwater resources less the overlap between the two. There are various hierarchies for the quality of natural water resources, a small portion of which is fresh water, and an even smaller portion of fresh water is available for us [1]. Based on the statistics done by National Bureau of Statistics of China, we take a value of 40% as the average portion of natural water resources as available for usage[5].

**2.2 The Grey Model**

We use the GM(1,1) model to predict water demand and supply in 2025. For every province, denote historical water demand and supply by:

where represents water demand or supply in 1999, and represents historical data of water demand or supply in 2011. Under the rule of accumulated generation operation, we get:

Averaging the sequence, we get a vector with 12 elements:

where Establishing the grey differential equation:

(1)

yields the albino differential equation for Eq.(1):

(2)

Solving for Eq.(2) yields:

The method is given in the book Application of MATLAB in mathematical Modeling (Zhuo and Wei, 2011) [6].

After getting estimated water demand and supply for each province in 2025, we calculate the water gap as:

and use water gaps for our further study.

**2.3 Prediction Results**

**Figure 2** shows the results by running MATLAB. Fifteen provinces will suffer from water shortage in 2025, namely Jiangsu, Xinjiang, Anhui, Shanghai, Henan, Hebei, Heilongjiang, Ningxia, Inner Mongolia, Gansu, Shanxi, Shandong, Hunan, Beijing and Tianjin, most of which are located in northern areas of China, except Shanghai and Hunan Province. The result is in agreement with the fact that water issue is more severe in northern China than in southern China, due to less precipitation, drier climate and more demand by agriculture, industry and urban consumption. More specifically, with additional 58.324 billion needed, Jiangsu Province ranks top for water shortage, followed by Xinjiang Province and Anhui Province, with 36.05 billion and 34.93 billion respectively (see **Figure 3**).

As can be seen in **Figure 2**, nearly half of China will be endangered by water shortage in 2025. Based on this prediction, in the following four sections, we propose four strategies to solve the problem.

Figure 2: Predicted gaps between water demand and supply across China in 2025

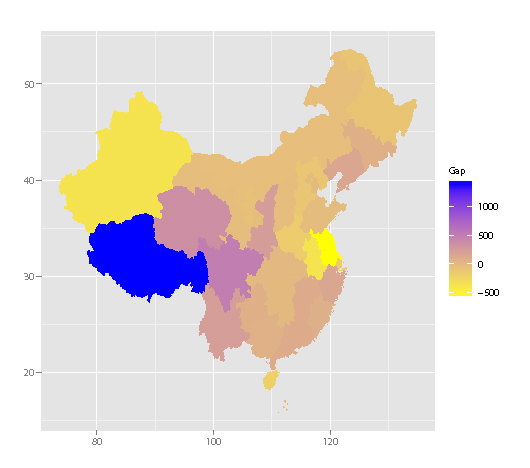
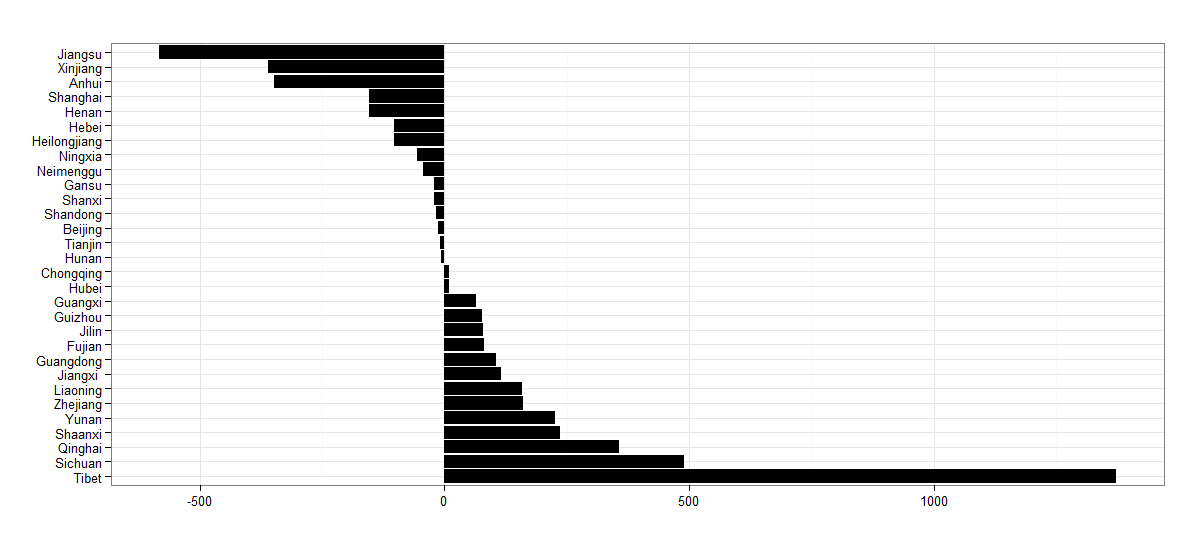


Figure 3: Predicted gaps water between demand and supply across China in 2025

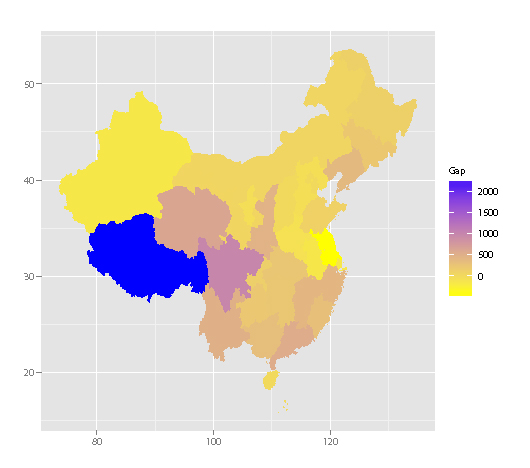
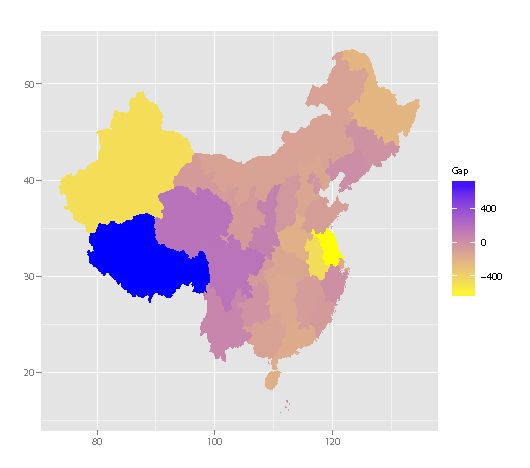


**2.4 Sensitivity Analysis**

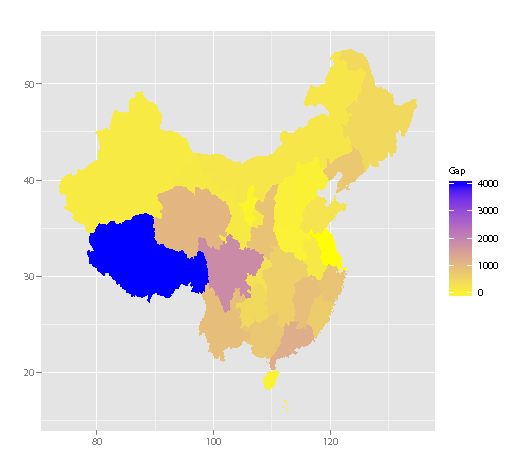
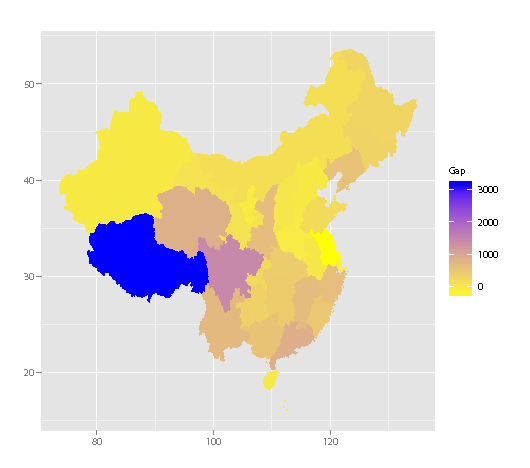
In our assumptions, we mentioned that a portion (40%) of water resources is available for usage. In this part, we try to test the model's sensitivity to changes in this number. **Figure 4** illustrates results of the analysis. It is suggested by the figure that the change barely affects our predicted water distribution across the country. The value of water gaps, however, changes to some extent with different portions (absolute values of water gaps is shown by the legends in the figure).

One flaw of the grey methods, as well as several models we mentioned before, is that we base the prediction on historical data trends and the assumption that this trend will continue in the future. It is never possible to accurately evaluate whether the assumption holds, so our model is still subject to possible error. A conceptual mode is offered by David et al. (1998)[7], which acknowledge the difficulty in quantifying water shortage prediction and base itself on dynamic system of water supply and demand and only requires a set of data in one single year. The authors discussed the precision of the model in their report. However, due to lack of required data, we give up on using their model. When related data are available, we suggest considering their model. More issues of data precision are discussed in the part of strengths and weaknesses of model.

Figure 4: Predicted Water Distribution on Different Portion of Available Water Supply



(a) 20% (b) 60%



(c) 80% (d) 100%

**3 Strategy 1: Water Transfer**

Given the estimated water conditions in 2025, we use a mathematical model to come up with a strategy for water transfer.

There are several major rivers in mainland China, and scholars often partition China into regions around these rivers. These regions are often referred to "river basins". An illustration of this partition is shown in **Figure 5**. For simplicity, we refer to river basins as "regions" for the rest of this part.

Since water transportation within a region is relatively easy and costless compared to across regions, we now only consider water transportation across regions. That is, we calculate water gap of every region by summing up that of every province belonging to the region.

Firstly, for every region, we take its water supply and demand as given, and figure out a model to determine an optimal transportation strategy satisfying the demand of every province. Based on this model, we input related data and get the desired strategy.

Figure 5: An illustration of river basins in China. Source: *Atlas of Natural Disaster System of China* (Shi et al., 2003) [8]



**3.1 Assumptions**

**Cost of transportation is proportional to the volume and the distance of water transported.** That is, the more and the longer distance water is transported, the more it costs for the government. It is unrealistic to transport a very small amount of water across regions, so we assume that the volume of water transported is large enough to ignore economies of scale.

**Transportation is accomplished at the beginning of a year.** We simplify a continuous water transfer to an event accomplished at the beginning of a year. After transportation, water demands in China are met to the largest extent.

**There exists a water transportation channel (or other water transfer projects) between every pair of regions.** There are several cross-regional water transfer projects built in China. A famous example is the South-North Water Transfer Project. We assume that under the help of these water transfer projects we are able to transport water between regions.

Notations used in the model are listed as follows:

Table 1: Notations Used in the Transportation Model

|  |  |
| --- | --- |
| Notation | Explanation |
|  | Region i. There are at total M regions involved. |
|  | Water supply of region i at the beginning of a year. |
|  | Water demand of region i for the year. |
|  | Water gap of region i. |
|  | Cost of transportation for per unit of water from region i to region j. |
| *E* | The set of net water suppliers, i.e., regions with water excess. |
| *S* | The set of net water demanders, i.e., regions with water shortage. |
| *m* | Number of elements in E, i.e., number of net water suppliers. |
| *n* | Number of elements in S, i.e., number of net water demanders. |
|  | The volume of water transported from region i to region j. |
| *C* | Total cost of water transportation. |

**3.2 The Transportation Model**

We first decide upon the set of regions with water excess or shortage by checking their gaps 's. We let if and let if . For legibility, we rename a positive gap from to and negative ones from to . We discuss below different cases corresponding to different values of total gap .

*In the case of G=0, total water excess equals to total water shortage.* The optimization problem can be listed as follows:

s.t.

*In the case of G>0, total water supply exceeds total water demand.* Therefore, everything else being equal, the first constraint changes to:

*In the case of G<0, total water demand cannot be satisfied by water supplies inside the country*. Therefore the second constraint changes to:

**3.3 Solution**

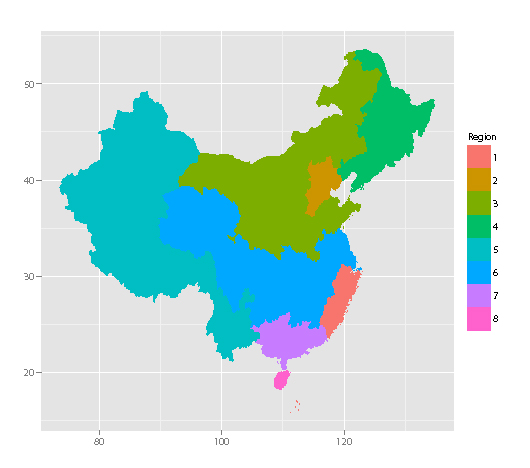
3.3.1 Parameter Estimation

*Regions.* As is shown in Figure 6, we use one of the mainstream partitions which divides mainland China into 7 major regions. We specify them in the Table 2 below.

Table 2: Classification of Provinces

|  |  |  |
| --- | --- | --- |
| Notation | River Basin | Provinces |
|  | The Southeast River Basin | Fujian, Zhejiang |
|  | The Haihe River Basin | Beijing, Hebei, Tianjin |
|  | The Yellow River River Basin | Gansu, Henan, Inner Mongolia, Ningxia, Shaanxi, Shandong, Shanxi |
|  | The Songliao River Basin | Heilongjiang, Jilin, Liaoning |
|  | The Southwest River Basin | Tibet, Xinjiang, Yunnan |
|  | The Long River River Basin | Anhui, Chongqing, Guizhou, Hubei, Hunan, Jiangxi, Jiangsu, Qinghai, Shanghai, Sichuan |
|  | The Perl River River Basin | Guangdong, Guangxi |

Figure 6: The 7 Regions We Use in the Model (Note that the region in the graph is Hainan Province, which we ignore in our model.)



*Supplies and demands.* We get from section §2.3 the projected water condition in 2025. According to our estimation, two regions will face water shortage, namely (12.32 billion ) and (5.21 billion ), and the rest 5 regions have water surplus (26.18 billion for , 15.90 billion for , 128.68 billion for , 53.37 billion for , 19.70 billion ). Total water gap in China will sum up to G=179.27 billion . The predicted net supplies and demands are also available, using the data above. In this case, we have E={, , , , }, and S={, }.

*Cost of transportation per per km.* We first determine the distances between regions. Since shapes of regions are irregular, there is no way to accurately capture the intrinsic distances, which leaves us to estimate such values. We take mean distance between provinces of each region as a measurement of region distances. To approximate this measurement, we choose a central city in each region and take the distance between these cities. We take cities Wenzhou, Beijing, Yan'an, Changchun, Lasa, Chongqing and Foshan to represent the center of from to , respectively (see Fig. 7). The distances calculated are listed below in Tab. 2. For example, the element in the first row and the first column stands for the distance between and . For simplicity, we arbitrarily assume that transporting water 1 km costs 0.1 yuan, then the unit cost equals to 10% of the distance between regions involved.

Figure 7: Cities Standing for Centers for Regions and an Illustration of Water Channeling Network

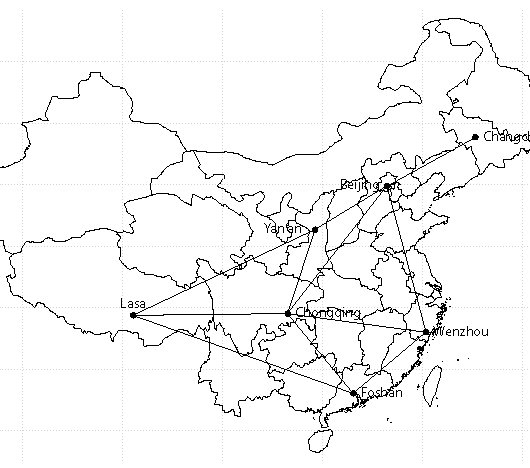


Table 3: Distance Matrix for Regions

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Distance (in km) |  |  |  |  |  |  |
|  | 1382 |  |  |  |  |  |
|  | 1420 | 710 |  |  |  |  |
|  | 1809 | 858 | 1564 |  |  |  |
|  | 2880 | 2564 | 1872 | 3400 |  |  |
|  | 1390 | 1460 | 828 | 2300 | 1490 |  |
|  | 940 | 1904 | 1550 | 2570 | 2310 | 980 |

The problem is a linear programming problem, or more, a typical transportation model. With costs, supplies and demands available, we specify the model and solve for the problem. The result is shown in Tab. 4.

3.3.2 Conclusion of Results

Based on the solution given by solving for the problem, we have a water channeling strategy to deal with water shortage in the year of 2025:

Transport 12.32 billion water from (The Songliao River Basin) to (The Hehai River Basin), which costs 1057.30 billion yuan;

Transport 5.21 billion water from (The Long River River Basin) to (The Yellow River River Basin), which costs 431.46 billion yuan.

The total cost sums up to 14.88 billion yuan.

Table 4: Water Channeling Strategy

|  |  |  |  |
| --- | --- | --- | --- |
| From | To | Volume | Cost |
|  |  | 12.32 | 1057.30 |
|  |  | 5.21 | 431.46 |
|  |  | Total Cost (C) | 1488.76 |

**3.4 Sensitivity Analysis**

The model uses a lot of estimated data as parameters, in addition to the estimated water gaps given by the prediction model. Therefore, we should carefully examine the model's sensitivity to changes in estimated results. Using the data given above as a basis, we look into the effect of partial changes of data. From **Table 5** we see that the total cost does not change more than estimation. But still, our model is to some extent reliant on the values of estimations. However, the optimality of our result is not negated by this sensitivity, since the model yields an analytic optimal result. In other words, our model always produces the best transportation path given that the related data are correct.

Table 5: Sensitivity Analysis of the Transportation Model

|  |  |
| --- | --- |
| Changes in distance between and | Change in total cost |
| 20.00% | 14.20% |
| 10.00% | 7.10% |
| -10.00% | -7.10% |
| -20.00% | -14.20% |

**4 策略2： 水存储**

和调水策略不同的是，水存储策略主要是为了解决水资源在时间分布上的不均匀，储存下来的水资源可备日后之用。常见的水存储方式包括自然的地下蓄水，以及人工水库等等。在这一节，我们主要解决如何利用水库来更好的解决未来水短缺的问题。

面对未来需求不确定，决定需要存储多少水资源非常类似于一个报童问题，即决定一个最优的订购量来满足未来不确定的需求，因此我们采用了经典的报童模型来刻画这一问题。为了叙述问题的方便，在这一部分，我们用“需求”这个词来表示之前提到的水缺口。

这一节剩余的部分，我们建立了一个理论的库存模型并以中国的三峡水库为案例对我们的模型进行了计算。

**4.1 假设与符号说明**

我们刻画的这一模型具有这样一些假设：

1. 水库的存储的水资源由上游水域供给、本地降水以及日常储水量构成。因此在决定水库水订购量时，水库所面临的水需求应为下游真实的水需求扣除掉水库本地的降水量以及水库的日常储水量；

2. 水库存储的水资源用来满足下游的水需求；

3. 由于储水不够，不能满足下游的需求，会为带来缺货成本。另一方面，如果储水过多，而不能为上游经济发展做出贡献，会带来机会成本；

4. 下游的水需求服从正态分布。其累积分布函数为F(x)，概率密度函数为f(x)。

本模型中所用到的一些符号说明如下：

**表6: 报童模型中的一些符号说明**

|  |  |
| --- | --- |
| 符号 | 解释说明 |
|  | 下游的总水需求，该变量服从正态分布 |
|  | 水库的订购量 |
| s | 缺货成本，反映在订水不足，下游的总水需求无法满足而带来的经济损失 |
| c | 机会成本，反映在订水过多，多余的水本可为上游带来经济利润而造成的机会损失 |
| TC(q) | 水库的总成本 |

**4.2 水库的报童模型**

我们从成本的角度建立了报童模型，以决定最优的订购量来最小化这一成本。具体的模型如下所示：

通过对TC(q)求一阶导数，并令其为0，我们得到了：

(3)

因为TC(q)关于决策变量q的二阶导数非负：

因此这一问题存在最优解使得总成本最小，通过求解方程（3），我们得到了最优订购量所满足的条件如下:

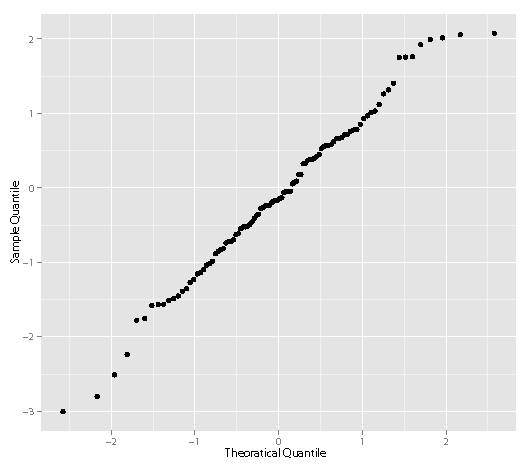
**4.3 案例研究：三峡水库**

坐落在宜昌市的三峡水库作为中国最大的水库，其代表性以及战略重要性不言而喻，是其下游省市的最大水资源提供方，包括湖南、湖北、江西、安徽、江苏以及上海。我们把理论模型应用到三峡水库上，借用实际数据，来决定三峡水库最用的订水量。

4.3.1 下游历史水需求数据的正态性检验

我们利用三峡水库下游各个省市的历史总水需求数据来来做正态性检验。图8所显示的Q-Q图说明了历史数据和正态分布之间有很好的吻合。Shapiro-Wilk检验也证实了我们的假设（W = 0.9885, p = 0.5429），即在样本数据符合正态分布的假设下，获得与观察值W=0.9885相等或更极端的值的概率是54.29%。因此，我们的数据支持我们的正态性假设，即我们认为下游的水需求数据服从正态分布。

**图 8: 历史书需求数据的Q-Q图**



4.3.2 参数估计

由于水资源短缺会对农业、工业生产以及城市消耗造成直接的负面影响，进而影响到当地的GDP，因此我们将缺货成本定义为由于下游的水需求没被满足而对下游GDP造成的损失。官方目前使用万元GDP耗水量来衡量水对当地GDP的贡献值，因此我们在模型中用下游地区每立方米水对GDP的贡献值来作为我们的缺货成本。

另一方面，如果水库订水过多的话，多余的水本可以对上游地区的农业、工业生产以及城市消耗带来经济价值，因而会产生机会成本。同样地，我们在这里使用上游地区每立方米水对GDP的贡献值作为模型当中的机会成本。并且简化起见，我们并不考虑多订的水所额外产生的管理或者环境成本。

三峡水库的下游地区包括湖北省、湖南省、江西省、安徽省、江苏省、上海市，其上游地区包括青海省、四川省、贵州省以及重庆市。缺货成本s用三峡水库下游地区各省市单位立方米水对GDP的贡献值的平均值表示，机会成本c则用三峡水库上游地区各省市单位立方米水对GDP的贡献值的平均值表示。我们从国家统计局[5] 获得各个地区近三年的GDP以及需水量，通过简单地计算，我们得到了缺货成本s=53.44元/立方米，机会成本c=222.43元/立方米。

将三峡水库当地的降水量以及日常储水量（水库的最低储水要求）考虑进来，我们将三峡水库下游的净需求（均以一年的跨度说明）定义为：

净需求 = 下游地区水缺口 三峡水库降水量 日常储水量

上面式子中的水缺口是相应下游省市总水缺口的平均值，其值经过计算为1544.8 亿立方米。三峡水库降水量按照下述公式进行计算：

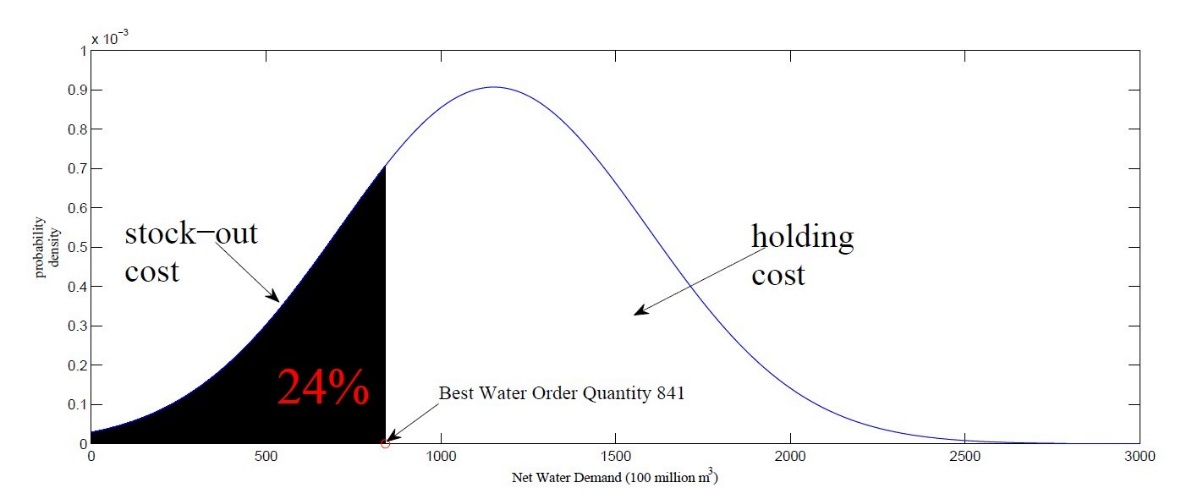
宜昌市的年均降水量数据从维基百科获得，同时我们也从维基百科查到了三峡水库的日常储水量为393亿立方米。

通过以上计算，我们得到净需求为1151.6亿立方米。因为我们将年降水量以及日常储水量视为常数，因此净需求的标准差即为水缺口的标准差，即440亿立方米。

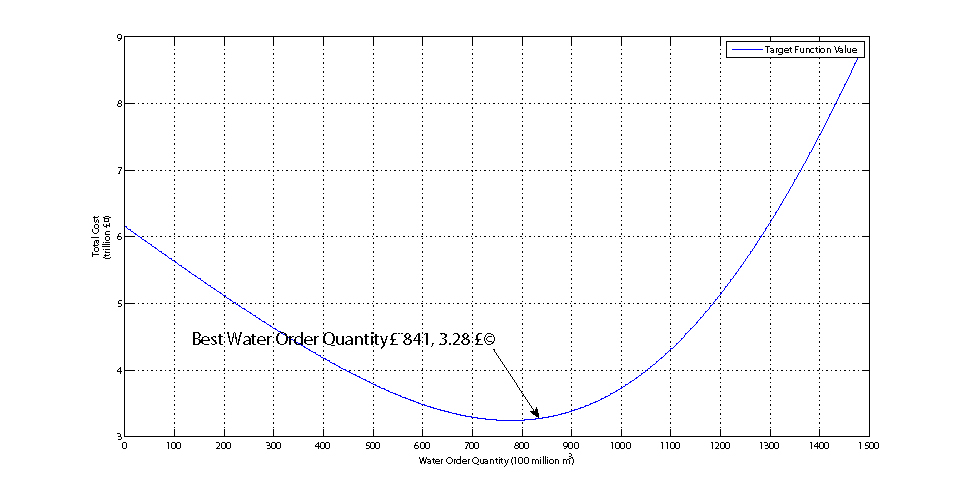
4.3.3 模型的仿真以及结果

有了净需求的分布函数以及缺货成本、机会成本的具体数值，我们便可以求解理论的报童模型。通过计算，我们得出三峡水库最优的订购量为841亿立方米（见图9）。我们通过遍历所有可能的订购量，用Matlab来计算相应的期望成本（见图10），可以看到当订购量为841亿立方米时，期望成本最小。与此同时，我们用均值为1152亿立方米，标准差为440亿立方米的正态分布随机生成了500个三峡水库下游地区的净需求量，并将最优订购量带入成本函数，计算在最优订购量下净需求的变动对期望成本的影响（见图11）。

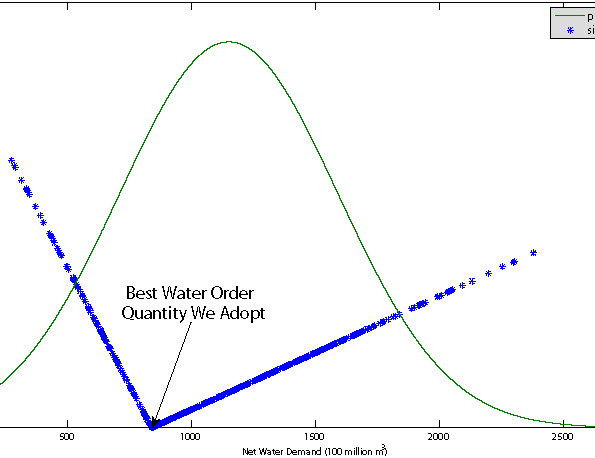
考虑2025年作为下一次订水期，我们建议三峡水库应该从上游地区订购841亿立方米的水资源来满足下游地区未来的水缺口。本案例也说明了报童模型是一种很强的解决实际问题的理论模型。通过获得更加精确的数据以及加入更多考虑因素进来，政府部门可以在三峡水库订购问题上做出更加明智的决策出来。

**图 9: 报童模型的求解**

**图 10: 报童模型期望成本函数**



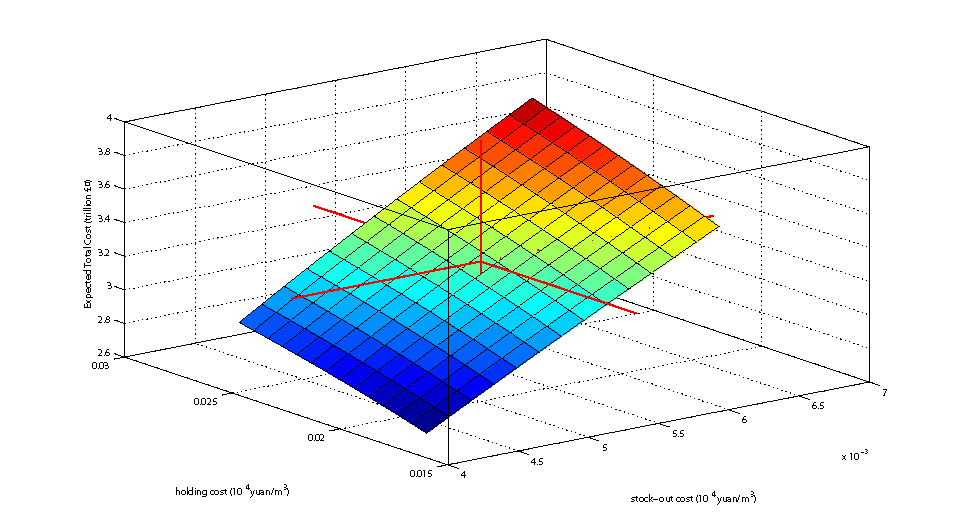
**图 11: 净水需求的蒙特卡洛模拟**



**4.2 敏感性分析**

通过我们模型计算得到的最小期望成本以及最优订购量可以帮助决策制定者更好的应对未来的不确定性。然而，诸如环境或者社会成本（环境破坏、强制性迁居）等因素并没有考虑进模型中。另外一个局限便是下游净水需求必须严格满足正态分布才可以。在我们的模型中，我们分别用上下游地区单位立方米水对GDP的贡献值作为我们对缺货成本以及机会成本的估计。由于各个省市其水对GDP的贡献度每年会有一定波动，下面，我们通过对缺货成本和机会成本做一定范围的改变，以看其对期望成本的影响（见图12）。

**图 12: 敏感性分析的可视化**



**5** **策略3: 海水淡化**

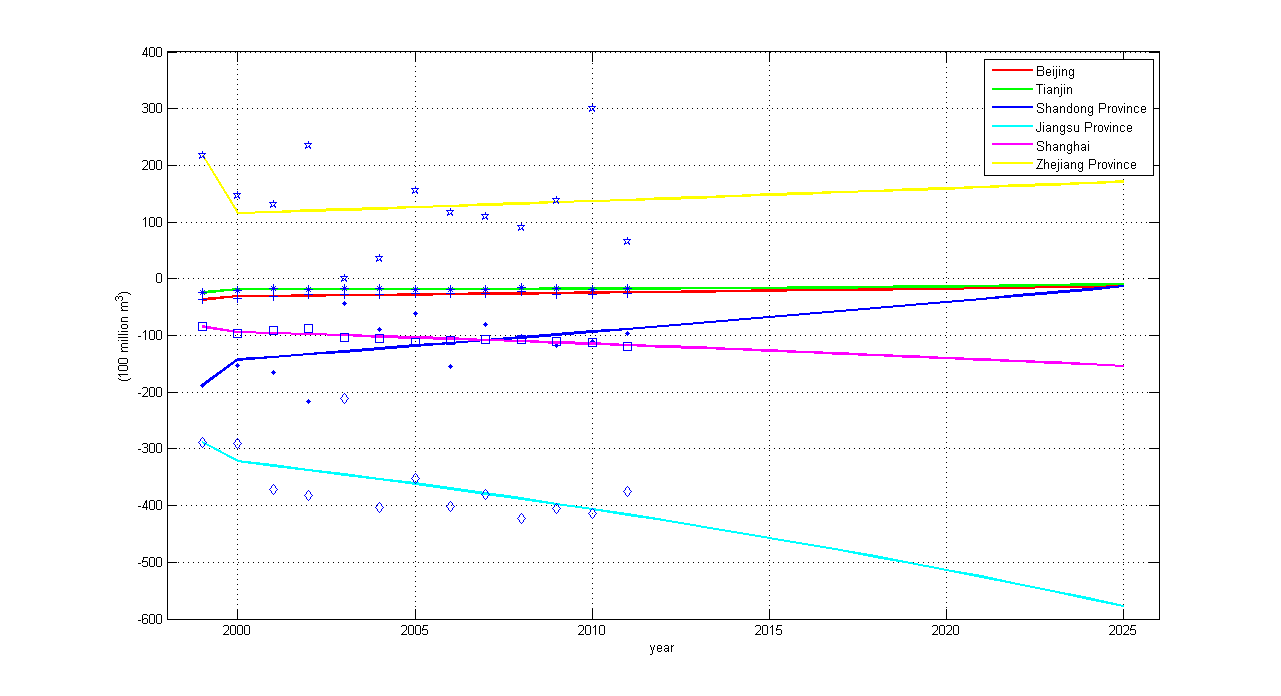
海水淡化是指通过蒸馏等方法降低海水中的盐和其他矿物质含量，来为人们提供淡水的一种方法。咸水毫无疑问是水供给的一个巨大来源，因此部分沿海地区采用了海水淡化来解决水短缺的问题，比如说沙特阿拉伯[9]。

在中国，天津市正在运营一家海水淡化厂来缓解当地严峻的水短缺问题，但是海水淡化并没有广泛的推广到中国其他地区。在这里，我们应用了净现值分析的方法去研究建立海水淡化厂的成本以及收入，之后决定在缺水地区是否要建海水淡化厂，如果建的话，需要建几个。

**5.1 海水淡化厂建立的潜在地区**

由于海水淡化本身的特点，海水淡化厂目标选址被限制在了沿海缺水地区。通过我们在第2节的预测，我们将目标选址区锁定在了北京、天津、上海、江苏省以及山东省（见图13）。

**图13: 六省市地区水缺口比较**



假定技术以及地理上建立海水淡化厂都是可行的，下面我们主要将精力放在海水淡化厂所带来的经济以及社会效益是否超过所投入的成本上。

**5.2 假设**

* **海水淡化厂作为所在地水缺口的唯一水资源提供方。**
* **不同地区的海水淡化厂之间并没有任何差异。**每一个海水淡化厂均有相同的产能，初始投资，单位淡化成本以及运营成本。
* **建立一个海水淡化厂平均需要2年的时间。**
* **海水淡化厂所带来的社会效益用单位立方米水对GDP的贡献值来衡量。**水资源对农业、工业生产以及城市消耗而言非常重要，进而直接影响到当地的GDP。

**5.3 符号说明**

* : 第t年的第i个省的净现金流
* : 第i个省的单位立方米水对GDP的贡献值
* : 第t年第i个省的GDP值
* : 第t年第i个省的水缺口量
* : 为了满足水缺口，各个省市所需要建立的海水淡化厂的数量
* : 每个海水淡化厂的初始投资，我们设为20亿元人民币
* : 每个海水淡化厂的运营成本，我们设为1.47亿人民币
* : 每个海水淡化厂的年产能，我们设为50亿立方米
* : 单位海水淡化处理成本，我们设为0.015元/立方米
* 折现率，我们设为5.12%.

以上参数的估计是从*《桃园海水淡化厂计划概述》*[10]中获得。

**5.4 成本-收益分析**

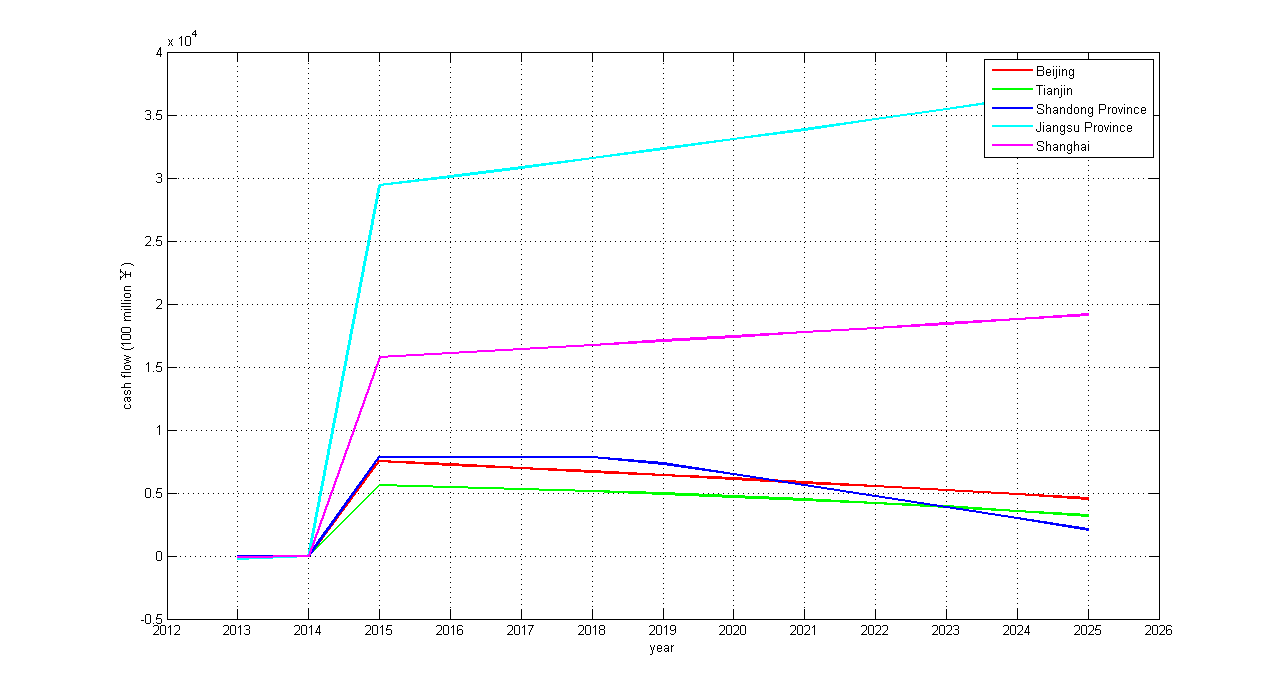
海水淡化厂建立的成本包括初始投资、运营成本、海水淡化处理成本，收入主要体现为水资源对当地GDP的贡献值上。海水淡化厂每一年的现金流如下所示：

其中 , 以及 是最近三年各个省市相应数据的平均值。通过将各个省市2025年水缺口量除以海水淡化厂的年产能，我们便得到了各个省所需建立的海水淡化厂数量n，其中北京、天津、山东各需要1个海水淡化厂，上海需要4个海水淡化厂，而江苏需要12个海水淡化厂来满足水缺口。

通过对海水淡化厂每年的现金流进行贴现，我们得到了投资海水淡化厂建立的净现值为：

通过我们的计算，结果说明对以下5个省（见图14）而言，建立海水淡化厂是能够增加社会效益的。特别地，江苏省因为采用海水淡化厂所获得的社会效益是最高的，其净现值为24.46万亿元。

**图14: 5省市海水淡化厂现金流图**



**5.5 策略说明**

如果建立海水淡化厂在技术上以及地理上是可行的话，我们强烈建议沿海缺水地区如果没有其他更好的应对水短缺的措施，可以考虑建立海水淡化厂来满足2025年的水短缺问题。然而，应该注意到目前天津市正在运营的海水淡化厂的实际产能很大程度上受到当地简陋的基础设施以及居民对淡化水需求的影响[11]。因此在决定采用海水淡化时，政府部门应该首先提高居民对淡化水的需求以及建立更好的基础设施。

**6 Strategy 4: Water Conservation**

Water conservation is as important as supply augmentation. In order to encourage citizens to conserve water and reduce waste, government has recently introduced such programs as increasing block tariffs, whose theoretical ground lies behind Ramsey Pricing. Through a case study of Shaanxi province, Liu and Gu [12] found that increasing block tariffs helps save about 15 water per person per year. So the projection seems quite promising to resolve water shortage problem in the future. Here, we focus on how to determine optimal increasing block tariffs, including water price and block cutoff.

Central to Ramsey Pricing is that the price markup should be inverse to the price elasticity of demand multiplied by a constant lower than 1 [13] . Scholars have applied the theory into public service sectors, such as water sector, where government hopes to maximize social welfare rather than profit [14, 15] . So, we also use Ramsey Pricing model to propose optimal water price for different income consumers.

**6.1 Assumptions**

* **We partition each province into agriculture sector, industry sector and urban sector and consider optimal pricing in each sector independently.** Agriculture, industry and urban consumption are three major drives for water use, of which the respective percentage is 61%, 24%, 13% [5] .
* **Based on different income owned by consumers in each sector, we partition each sector into low income, medium income and high income.** Three blocks are used extensively both in theory and practice, because too many blocks are difficult to implement while too few blocks are ineffective in water conservation [16] .
* **Demand in each block is endogenous, which can be adjusted by water price.** Classic economic theory states that price is a strong tool to affect demand.
* **We consider government as the water supplier firm and the marginal cost and fixed cost remain the same for different groups in each sector.**
* **The government only requires that total revenue equal to total cost.** Unlike corporations, water supplier firm pays more attention to social welfare. Even it does generate profit, the profit is usually quite low [17] .
* **Pricing strategy is the same for three sectors.** For simplicity, the model presented below describes the strategy in one sector and can be applied in the other two.

**6.2 Notations**

* : Demand for different income groups, where i can be low, medium or high.
* : Price for different income groups, where i can be low, medium or high.
* : Demand elasticity for different income groups, where i can be low, medium or high.
* : Intercept in demand equation for different income groups, where i can be low, medium or high.
* : Total cost faced by the government.
* : Marginal cost faced by the government, which remains the same in each sector.
* : Fixed cost faced by the government, which remains the same in each sector.
* : Total revenue earned by the government.
* : Marginal social welfare for different income groups, where i can be low, medium or high.
* : Social welfare for each sector.

**6.3 Model**

Bailey [18] used linear regression and double logarithm linear regression to describe the demand based on water price and found that the latter fits the data better. Align with Bailey's finding, we also adopt double logarithm linear regression, which can be written as:

whereε , representing the demand elasticity for each group in one sector.

For the government, total cost can be written as:

and total revenue cost can be written as:

With one unit of water increased, consumers pay , and government pays MC. So the marginal social welfare can be written as:

We aim to determine an optimal demand to maximize total social welfare. The problem can be written as:

To get optimal demand, we use the method of Lagrange multipliers to get optimal price for each consumption group first. The optimization problem above can be rewritten as:

Taking first-order derivation of equation (6) to zero, we get:

As a result, for each consumption group, we have:

Expressing and in terms of , we get:

Plugging equation (7) and (8) into equation (4) and (5) and making total revenue equal to total cost, as we assumed, yields:

where,. Solving the equation \ref{e16}, we can achieve the optimal price for different amount of water consumption and further the optimal demand, which can be used as cutoff for the increasing block tariffs.

**6.4 Example Analyis and Strategy**

Agricultural sector accounts for the most in the total use, but only recently do some provinces, such as Hunan province, begin to charge irrigation cost due to difficulties in implementing. So it might be premature to adopt increasing block tariffs in agricultural sector.

Introduced into urban sector recently, increasing block tariffs has received extensive discussion in China. Considering optimal strategy in urban sector, we apply an example analysis to test its validity.

Demand elasticity is roughly between -1 and 0 according to the basic microeconomic theory. Jia and Kang [19] found that the figure is -0.346 in China. So we assign -0.4 to . In our model, low water consumption can be regarded as minimum requirement for water demand with a small elasticity of -0.7. The same logic applies to high water consumption group and we assign -0.1 to . For other parameters, we assign value subjectively (see Table 7).

Table 7: Notations

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |

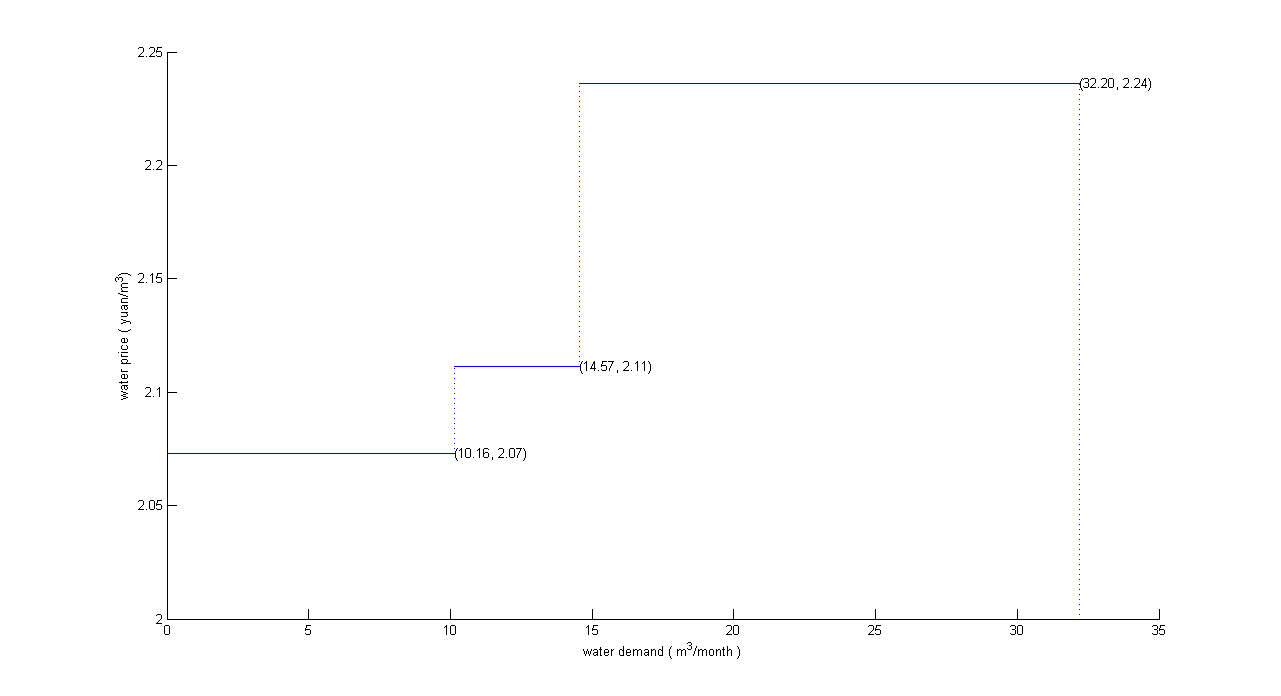
The results, which are summarized in table 7 and figure, show that when the consumption below about , the optimal charge should be set at . When the consumption between and , the price should be . When the consumption exceeds . The results fit with common sense, but more thorough investigation should be taken to get more accurate value of parameters, which are essential for the final pricing policies.

Table 8: Example Analysis: Optimal Pricing Strategy

|  |  |  |  |
| --- | --- | --- | --- |
|  | First Block | Second Block | Third Block |
|  | 10.16 | 14.57 | 32.2 |
|  | 2.07 | 2.11 | 2.24 |

Depending on the data provided by Liu and Gu [12] , we apply the results above to Xi'an city and obtain water saving per person per year, even more effective than the existing plan, indicating the strength of our method. However, thorough investigation should be taken to get more accurate estimations of parameters, which are essential for the final pricing policies, but we believe it is a strong tool to help government make wise decisions.

Figure 15: Example Analysis: Optimal Pricing Strategy



**7 A Comprehensive Strategy**

We now aim to synthesize four strategies discussed above into a comprehensive proposal for decision makers. We believe that each strategy has its pros and cons, which are summarized in Figure 16, and the government should adjust measures according to local conditions when addressing water shortage.

**Water transfer** is advantageous in emergency situations, and is especially useful in dealing with uneven spatial distribution of water resource. However, water transfer project incurs a great amount of money and takes a long time to construct. **Water storage** is easy to implement in the sense that reservoirs are located near large rivers and enjoy easy access to water. The location of a reservoir, however, is also its limitation since it might also bring such negative impacts as too much pressure imposed on ecological environment. The strategy applies best when downstream demand is relatively stable, i.e., no major unexpected emergency arises. **Desalinization** produces water of higher quality, making it a powerful tool in regions that are lacking clean, usable water. While the strategy has almost unlimited source of water and can produce by-product such as salt, its cost relies heavily on related technology. Desalinization can only be applied in coastal regions now and we also recommend that it be adopted in developed cities first because it requires related infrastructure and market of drinking water. **Increasing block tariffs pricing** can be widely used in cities, with ease of implementation and relatively low cost. But the market response, reflected in demand reduction, might have time lag.

Based on two criteria, which are inland area *versus* coastal area and uncertain water demand *versus* certain water demand, we can categorize four strategies into two dimension coordinate, as seen in Figure 17. We define level of demand certainty as variations of historical data, which can be accessed by the department concerned.

Combined with our prediction of water gap, we propose 4 specific plans now to deal with the problem in 2025.

1. Priority should be given to **Jiangsu province** since it will suffer most from water shortage. Jiangsu is a relatively developed province, so we recommend **water storage, 5 desalinization plants and increasing block tariffs**. Many reservoirs are located in Jiangsu, department concerned should pay attention to make stock policy based on our news-vendor model. 5 desalinization plants are suitable considering other strategies to cut water gap. In fact, IBT has been introduced into Nanjing city and Nantong city, and we highly recommend that it be promoted across the province by using our model provided. Investigation should be careful carried out to estimate parameters precisely.
2. Strategic position as it is in China, **Shanghai** also should be on top lists. Like Jiangsu province, Shanghai is a highly developed city along the coast and it is characteristic of scarcity of clean water, so we recommend **1 desalinization plant and increasing block tariffs** Considering high purchase power of consumers in Shanghai, we predict that the price set at large water consumption level should be higher than that in other cities, if our model is used correctly.
3. **Henan** is representative of inland area which mainly depends on agriculture, so we recommend **water storage** to solve the problem since the mainstream of Yellow river flows through He'nan province. Decisions should be carefully made on how much water needed to store for later gap. We believe our news-vendor model can help government make wise decision.
4. For **Beijing, Tianjin and Shandong province**, we recommend **1 desalination in each region, water transfer from southern area and increasing block tariffs.** Three neighbouring provinces can generate great synergy. Typical of scarce water resource, northern China is direct beneficiary of water transfer project.

Figure 16: Pros and Cons of different models

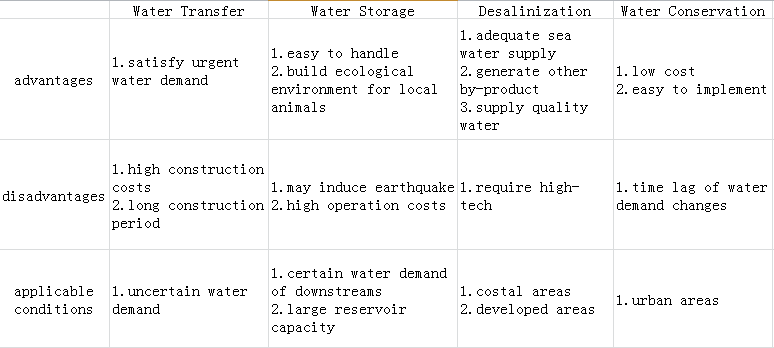
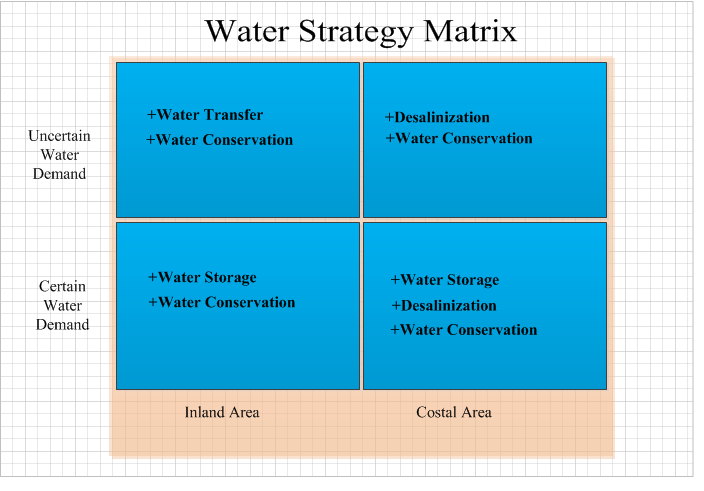


Figure 17: Categorization of four strategies



**8 Conclusion**

**Q1: What is the estimated water demand and available water supply in 2025?**

Based on grey model, we predict that **15** provinces will be in short of water in 2025, most of which are located in northern China. Jiangsu province will be most endangered by water shortage, with a gap of 58.32 billion yuan.

**Q2: How to solve the foreseen gaps?**

Considering spatial distribution, we apply a transportation model to address water transfer. Partitioning the vast country into seven river basins and summing up water gaps of provinces belonging to the region, we find that the Haihe Basin and the Yellow River Basin will be short of water in 2025. Assuming transport cost is proportional to distance, we get the optimal transfer strategy in which the Songliao Basin transfers water to the Haihe Basin and the Long River Basin to the Yellow River Basin, with a total cost of 14.88 yuan.

Considering temporal distribution, we apply a news-vendor model to address water storage. Regarding reservoirs as distributors who order water from its upstream to satisfy demands of its downstream, we aim to figure the optimal level of storage reservoirs need to prepare for future use. A case study of Three Gorges Reservoir reveals that should be stored to meet the demand of its downstream, including Jiangsu, Anhui and Shanghai.

Considering supply augmentation, we apply a NPV method to determine whether to establish desalinization plant. We first narrow down the potential locations to 5 provinces that will suffer water shortage in 2025 along the coast and obtain the number of plants needed under the assumption that water gaps must be satisfied. Upon reasonable assumptions about costs and benefits, we find that NPV of the project in each province is positive, indicating that desalinization is economically and socially promising to solve the water shortage.

Considering demand constraint, we apply Ramsey pricing model to address water conservation. We aim to propose an optimal increasing block tariffs based on different water consumption. Through a relatively subjective example analysis, we demonstrate the validity of the model, but more precise estimation of parameters are needed to get the final result.

**Q3: How to implement four strategies properly?**

Four strategies have their own pros and cons, so we should take measures according to local conditions. In order to provide a succinct and clear guide for government to make decisions, we categorized strategies according to two criteria, which are inland area versus coastal area and uncertain water demand versus certain water demand. Desalination and increasing block tariffs should be adopted in cities rather than rural areas.

For those which will face water shortage in 2025, we propose detailed plans. More specifically, we recommend Jiangsu adopt water storage, desalination and increasing block tariffs, Shanghai adopt desalination and increasing block tariffs, Henan adopt water storage and Beijing, Tianjin, Shandong combined adopt desalination and water transfer.

**9 Strength and Weakness**

One major problem facing us is the precision of data. Data from different resources follow different criteria, thus may present inconsistency overall. Also, although our data come from official sources like the National Bureau of Statistics of China, they are still subject to manipulation for many reasons. Different interpretations of data, on the other hand, lead to different result. Other data used for our parameter estimation, for example, per unit cost of water transportation, cost of building an desalinization plant and so forth is also hard to attain or estimate, these will greatly result in significant changes in final strategy should our estimation deviates from the intrinsic value.

However, to obtain the desired data is no easy job. These often require long-term surveys and study, as well as the assistance from expertise in related field. In several days, there is no way to accurately capture these data. Acknowledging this fact, we manage to build conceptual models with logical reasoning and mathematical calculation (The grey model is an exception. The reason for using a grey model is discussed in the section of prediction and an alternative conceptual model is also offered.), under the assumption that we have a precise data. This way, we are able to modify our final strategies as soon as we obtain more accurate data, say, from the government or other sources, without doing many burdensome repetitions.

**10 A Non-technical Position Paper to Governmental Leadership**

To Whom It May Concern,

We are writing this position paper to suggest a best strategy which combines desalinization, conservation, movement and storage to help you tackle with projected water shortage facing China in 2025.

We estimate that situation will be quite severe in 2025. There will be about 15 provinces whose water needs cannot be covered by local water supplies. Jiangsu province, in particular, will be faced with 58.32 billion of water shortage. Beijing, Shanghai and several other provinces also will be short of fresh water.

We suggest that in year 2025, the best water transfer strategy is to transport 12.32 billion of water from the Songliao Region to the Haihe Region, and 5.2 billion of water from the Long River to the Yellow River. On the other hand, our study on the Three Gorges Dam suggests that the Dam should store 84.1 billion of water from upstream to satisfy water needs in its downstream. For cities desalinization plants should be built in Shanghai, and several more in other provinces in need of water. The example of Shaanxi province tells us that an increasing block tariff pricing policy can well constrain water demands.

More specifically, in order to tackle with the severe water shortage in Jiangsu province, we suggest an increasing block tariff pricing policy combined with construction of 5 desalinization plants. Using more efficient storage strategy in water reservoirs can significantly generate stable water supplies to meet the demand in the province.

Our methodology combines four strategies together to deal with water shortage in a complicated environment in China. Jointly, these strategies cover demands of different degree of urgency, and different geographical distribution. We are confident that under various situations, our strategy will yield an optimal solution for you.

However, a major problem we encounter in our study is the precision and creditability of data. Since our models are conceptual ones using logical reasoning and mathematical computation, we are able to modify our final strategies as soon as we obtain more accurate data. Therefore, if you are interested in our study and provide us with field expertise and more credible results, we are more than happy to determine an even more detailed and effective strategy. We hope that with our joint effort, we can best alleviate water shortage and fight for a bright for the Country.

Best regards,

Team #17444

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